

# Long Range Forces in Quantum Gravity

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## Abstract

We calculate the leading quantum and semi-classical corrections to the Newtonian potential energy of two widely separated static masses. In this large-distance, static limit, the quantum behaviour of the sources does not contribute to the quantum corrections of the potential. These arise exclusively from the propagation of massless degrees of freedom. Our one-loop result is based on Modanese's formulation and is in disagreement with Donoghue's recent calculation. Also, we compare and contrast the structural similarities of our approach to scattering at ultra-high energy and large impact parameter. We connect our approach to results from string perturbation theory.

## I. INTRODUCTION

Quantum gravity is a subject beset with calculational difficulties, most predominantly due to ultraviolet divergences that are incumbent in any 4-D theory with a coupling constant with dimensions of length. These difficulties are well documented in the literature and have not allowed a consistent formulation of the theory at short distances [1]. The hope is that superstring theories will provide us with a robust definition of the theory in that regime. However, as Donoghue [2] recently pointed out, one can analyze quantized general relativity within the computational framework of effective field theory. He argued and showed that the unknown ultraviolet structure of the theory is irrelevant for the purposes of obtaining well-defined leading-order in  $\hbar$  quantum corrections to some quantities involving gravitational effects at large distances. It is possible to formulate this program in a general way armed with techniques based upon general analyticity properties and gravitational Ward-Slavnov-Taylor identities. As we will discuss, this strategy is also implicit in the analysis of the scattering problem at ultra-high energy and large impact parameter considered by a number of authors [3–5] in the past few years. The problem of long range forces is primarily a consequence of soft virtual gravitons and inherently of an infrared nature. The infrared region with respect to radiation of soft gravitons and the Block-Nordsieck [6] cancellation of infrared singularities was communicated long ago by S. Weinberg [7] and B. DeWitt [8].

Donoghue employed this method to obtain the leading corrections to the Newtonian potential. Our starting point is similar in spirit but quite different in implementation. We present and apply a general formalism presented recently by Modanese [9] which we believe is better suited for the study of the simplest quantity in the long distance regime, namely the static potential. We obtain a different result than Donoghue, and we discuss the reasons for the discrepancy in detail. In particular, we compute the quantum corrections in the gravitational or more generally the zero-mass sector of the theory in a manner that can be applied to either the static or ultra-relativistic problems. This is in parallel with studies of non-Abelian gauge theories. [10–13]. The approach of Modanese is presented in terms

of the Euclidean functional integral and parallels the Wilson loop of gauge theory. The formalism is quite general and allows a starting point for perturbative investigations of the long distance regime of this work as well as non-perturbative numerical simulations. On the other hand, the application to the relativistic scattering domain has been communicated in the literature under the names of relativistic eikonal, and semi-classical (WKB) techniques by many authors [14–16]. The two problems are very closely related and a general result can be derived by functional integral techniques which can be characterized by the exponentiation of certain connected Green’s functions. The connected nature of the exponentiation of the Green’s functions separates the potential from its iterations in a clear manner.

The actual work in such calculations is two fold. Firstly, one identifies a formulation that allows a convenient isolation of the effective theory in the relevant kinematic domains. In the static limit the relevant effective theory is a 3-D, dimensionally reduced quantum gravity, while in the ultrarelativistic domain, the effective theory is 2-D. Secondly, the isolation of contributions that survive as inverse powers at large distances, i.e. as  $R^{-n}$  for  $n > 1$  correspond to singular contributions at small momentum transfer i.e. analytic in the cut  $Q^2$  plane with branch cut singularities at zero momentum transfer. The Fourier analysis of the Newtonian potential ( $n = 1$ ) is already singular at small momentum transfer with the well known pole in  $Q^2$ . The specific power and nature of the singularities depends upon the number of graviton interactions and other details that will be elucidated in the following sections.

Contributions analytic at small momentum transfers (in the sense of old S-matrix theory) correspond to forces of finite range of the form  $\exp -(R/L)$  due to the Riemann-Lebesgue Lemma, and permit a Taylor series expansion at small momentum transfer. It is tempting to say that modern superstring theory will relate the range or the effective length parameter to the string tension in a way that all such contributions can be rendered finite. However, tempting such a correspondence is, its proof certainly lies outside the scope of this investigation. Nevertheless, we will expound on this idea using known one-loop results from string theory in a later section of this communication. The field theory limit with zero mass par-

ticles in loops arises from string interactions (higher genus Riemann surfaces), as required by unitarity. This limit is achieved at zero string tension, with polynomial corrections with finite coefficients which characterize the surviving string interactions at low energy. It is only the effective field theoretic (or S-matrix) contributions with zero mass intermediate states that reduce to non-polynomial interactions at low energies.

The organization of this communication is as follows: In Section II, the underlying technique pioneered by Feynman [17] and developed by DeWitt [8] and Mandelstam [18] is briefly reviewed. In this approach the metric is treated as a tensor field on a flat space-time background in parallel with any other quantum field. Path integral quantization is used as a convenient framework to introduce external sources to formulate the static potential (or scattering amplitude). The recent work of Modanese is presented, as well as mention of the older work of many authors on the high energy scattering. The relevant dimensionally reduced effective field theories are identified. A discussion of the results in terms of connected graphs of the gravitational and more generally the zero mass sector is presented. Section II also deals with the calculational aspects of this work, namely the relevance of the different distance scales and the isolation of the power behaved contributions at large distances. Calculations in the ultra-relativistic regime, relevant to the scattering of constituents at Planckian energies are reviewed but not presented in any great detail, since these have been dealt with in the past few years by a CERN group [19].

Following the explicit calculations, a general anatomy of a proof is presented in Section III to indicate how one would make a general proof based upon the plausible scenario that the small distance effects do not survive at large distances. Here recent calculations of higher order (higher genus surfaces) contributions in string theory are quite necessary.

Finally, we consider the pitfalls and limitations of such an investigation and compare and contrast gravity with other gauge theories, where long range forces play out in different ways. In abelian gauge theories the finite fermion mass screens all contributions other than the Coulomb potential between charged particles in the deconfined phase. There are also non-trivial, inverse-power behaved van der Waals forces between neutral objects resulting from

effective non-renormalizable interactions at low energies [20]. While the techniques in gravitational theory are similar to the analysis of molecular forces, the physics is vastly different. In non-abelian theories, confinement and spontaneous symmetry breakdown are plausible dynamical mechanisms that render long range inverse power behaved forces inoperable. An appendix deals with some technical questions on the details of the calculations.

## II. FORMULATION

### A. Static Limit

The general expression for the static potential energy for the gravitational interactions of two static massive particles requires some care and has been given recently by G. Modanese [9]. His work builds upon some expositions of other authors in both the static limit and the ultra-relativistic limits. Standard conventions will be followed with a flat space metric signature given by  $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ : we will work in  $M_4$  (4-D Minkowski space) instead of Euclidean space, since we will be interested in the perturbative definition of the theory with no applications to numerical simulations, or issues of instability. The Einstein-Hilbert action is given by

$$S_{\text{EH}} = -\frac{1}{16\pi G} \int dx \sqrt{g(x)} R(x), \quad (2.1)$$

where the integral is over the full four dimensional space time,  $g(x) = -\det(g_{\mu\nu})$  and  $R$  is the curvature scalar. It suffices to consider the action of two constituents, since we are considering the two body potential. The coupling of the sources to the metric is via

$$S_{\text{M}} = -M_1 \int ds_1 - M_2 \int ds_2, \quad (2.2)$$

where  $ds = (g_{\mu\nu}(x(\tau))\dot{x}^\mu\dot{x}^\nu)^{\frac{1}{2}}d\tau$  is the line element along the trajectory of each object of mass  $M_1$  and  $M_2$  respectively,  $\tau$  is the proper time measured along the trajectory, and  $\dot{x}$  is the covariant four-velocity. The generating functional for the gravitational interactions between two such massive particles is given by the functional integral:

$$Z_M = \frac{\int [Dg] \exp \frac{i}{\hbar} (S_{EH} + S_M)}{\int [Dg] \exp \frac{i}{\hbar} S_{EH}}. \quad (2.3)$$

The integration is over the metric  $g$  with some suitable gauge fixing for the perturbative definition of the Feynman diagrams of the theory. We seek the potential energy of two static sources situated at

$$x_1^\mu = (t_1, \vec{0}) \quad x_2^\mu = (t_2, \vec{R}), \quad -T/2 \leq t_i \leq T/2 \quad (2.4)$$

These trajectories correspond to two static particles with coordinate separation  $R$  and four-velocities  $\dot{x}_1^\mu = \dot{x}_2^\mu = \delta_0^\mu$ , with no relative motion. The limitations of the restriction to static sources within the context of the full dynamics is discussed below.

In the perturbative definition of the functional integral Eq. (2.3), the metric is expanded around the flat space metric  $\eta_{\mu\nu}$  in the usual fashion of the Feynman-DeWitt method of quantization of general relativity, namely

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x), \quad (2.5)$$

where  $\kappa^2 = 32\pi G$ . Following Symanzik [21], we shall identify the lowest energy of the system (the gravitational potential energy, in the static case at hand), with the limit

$$V(R) = \lim_{T \rightarrow \infty} \frac{i\hbar}{T} \ln Z_M, \quad (2.6)$$

in view of the identification of  $Z_M$  with the vacuum-to-vacuum amplitude in the presence of external sources, given by  $S_M$ , adiabatically switched on at  $-T/2$  and off at  $T/2$ ,

$$Z_M = \langle 0 | e^{-iHT/\hbar} | 0 \rangle = \sum_n e^{-iE_n T/\hbar} |\langle 0 | n \rangle|^2, \quad (2.7)$$

for a very large time  $T$ . It is clear that Eq. (2.6) above is meaningless as it stands. We give it meaning by endowing the large Minkowski time interval  $T$  with a small negative phase, or by keeping track of the  $i\epsilon$ 's in the perturbative propagators. This particular definition of the ground state energy was first advocated by Symanzik and proven in perturbation theory for the case of a source coupled linearly to the quantum field. It has been used in many other

field theory contexts. In the application to the static potential problem, this formulation clearly differentiates the static potential from its iterations, the higher order terms in  $T$  of Eqs. (2.6) and (2.7). We will assume that this definition holds even in the case presence of sources coupled to higher-order local monomials of the field.

If  $g$  is expanded as in Eq. (2.5) in Eq. (2.3), we obtain (to lowest order in  $\kappa$ ) the customary result,

$$Z_M = \langle \exp -\frac{i\kappa}{2\hbar} \int dx h_{\mu\nu}(x) T^{\mu\nu}(x) \rangle, \quad (2.8)$$

where  $T_{\mu\nu}$  is the classical energy-momentum tensor of the source, and averages with respect to the normalized gravitational functional integral are defined in the usual way,

$$\langle \mathcal{O} \rangle = \frac{\int [Dh] \mathcal{O} \exp \frac{i}{\hbar} S_{\text{EH}}}{\int [Dh] \exp \frac{i}{\hbar} S_{\text{EH}}}. \quad (2.9)$$

A trivial time dependence proportional to  $\exp -i(M_1 + M_2)T/\hbar$  may be absorbed into the normalization of the functional integral.

In the case of a pair of static heavy particles of masses  $M_1$  and  $M_2$  with trajectories given by Eq. (2.5), we have

$$S_M = -M_1 \int dt_1 \sqrt{1 + \kappa h_{00}(x(t_1))} - M_2 \int dt_2 \sqrt{1 + \kappa h_{00}(x(t_2))}, \quad (2.10)$$

and

$$T^{\mu\nu}(x) = M_1 \delta_0^\mu \delta_0^\nu \delta^{(3)}(x - x(\tau_1)) + M_2 \delta_0^\mu \delta_0^\nu \delta^{(3)}(x - x(\tau_2)). \quad (2.11)$$

We close this subsection with some remarks. In general the definition of the potential energy Eq. (2.6) and Eq. (2.9) appear to be frame-dependent and path-dependent in any field theory including gauge theory. The analogue in gauge theory, the Wilson loop, is gauge invariant but path-dependent unless the field strength tensor vanishes identically<sup>1</sup>. While the analogous expression in general relativity does not appear generally covariant

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<sup>1</sup>It is possible to construct a reparametrization-invariant “Wilson-loop like” quantity (see [9])

(gauge invariant), the results Eqs. (2.3) and (2.10) are still a fully covariant definition of the interaction in the static limit much in the same way that the Coulomb interaction is a necessary ingredient in the choice of Coulomb gauge in quantum electrodynamics for a consistent gauge invariant dynamics. Of course it will be necessary to consider kinetic energy and the full dynamics to establish the general covariance in a completely convincing manner.

The particular choice of trajectories Eq. (2.4) insures that the static potential alone is being calculated, and that no terms involving the kinetic energy or relativistic corrections are being included. At the level of  $(v/c)^4$  and  $V \cdot (v/c)^2$  relativistic corrections are bound to become important in the determination of motion in accord with the post Newtonian approximation of classical general relativity, where both terms of order  $GM/Rc^2$  and  $(v/c)^2$  are retained. The virial theorem of classical mechanics relates kinetic energy  $Mv^2$  to potential energy  $V$ . Therefore, kinetic energy will have to be included to order  $(v/c)^4$  in order to have a consistent dynamics for the purpose of computing the subsequent motion of this pair of gravitating objects. In our work Eq. (2.6) and the incumbent static sources Eq. (2.4) form a convenient starting point for the computation of the static potential. The last remark underscores the statement above concerning gauge invariance (general covariance).

Notice that in the weak coupling limit Eq. (2.8), the lowest-order energy is invariant under infinitesimal coordinate transformations i.e.

$$h_{\mu\nu}(x) \rightarrow h_{\mu\nu}(x) + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x), \quad (2.12)$$

for a conserved energy momentum tensor in Eq. (2.8). The complete static problem also has a partial covariance with respect to time independent coordinate transformations that are in the little group of  $g_{00}(x_i(t))$ , where  $x_i(t)$  are the trajectories, Eq. (2.4).

It is also worth mentioning that neither Eqs. (2.6) or (2.8) is a loop integral, since the notion of oppositely charged sources does not exist for a tensor theory of gravity. This property is essential in obtaining the Wilson loop for the static energy in gauge theories. One may also consider the possible physical meaning of a gravitational loop functional or holonomy,



$$\mathcal{U}(C) = -4 + \text{Tr} \mathcal{P} \langle \exp i \oint_C \Gamma_\mu dx^\mu \rangle, \quad (2.13)$$

where one views the connection coefficients  $\Gamma_{\mu\lambda}^\nu$  as matrix elements of a matrix  $\Gamma_\mu$ . This issue was investigated by Modanese [22]. He showed that the expectation value of the loop, calculated perturbatively around a flat background, does not give rise to long-range interactions, and so cannot be interpreted as representing a gravitational potential. The physical interpretation of such a result can be understood in terms of the functional integral. To leading order in  $\hbar$ , the vanishing of the holonomy implies that the weak field configurations which contribute to the functional integral have zero curvature (to that order). An intuitive way to understand this discrepancy between gravity and gauge theories is by noting that the coupling of the vector field to matter is in terms of  $\int d^4x A_\mu j^\mu$ . For a pair of oppositely charged static sources, this coupling reduces to the Wilson-loop exponent. For gravity, on the other hand, the coupling to a source is non-linear in the metric (see Eq. (2.10)). Even the perturbative expansion involves a coupling of  $h_{\mu\nu}$  with the energy-momentum tensor  $T_{\mu\nu}$  (see Eq. (2.8)). The affine connection coefficients simply do not arise in such an expansion.

We proceed next to the calculations of the first-order corrections in  $G$ , both semi-classical and quantum mechanical, to the classical Newtonian potential energy, using the formalism developed in the previous sections. The choice of trajectories in Eq. 2.4 ensures that the static limit is retained as opposed to calculating any effects associated with relativistic corrections. In the following the harmonic or de Donder gauge is maintained,

$$\partial_\mu (h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h^\lambda_\lambda) = 0. \quad (2.14)$$

In the harmonic gauge the graviton propagator is simply

$$D_{\mu\nu\rho\sigma}(x) \equiv P_{\mu\nu\rho\sigma} \Delta(x) = \frac{1}{2} (\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho} - \eta_{\mu\nu} \eta_{\rho\sigma}) \Delta(x), \quad (2.15)$$

where

$$\Delta(x) = -\frac{1}{4\pi^2} \frac{1}{x^2 - i\epsilon}, \quad (2.16)$$

is the free massless propagator. The choice of gauge restricts the form of Green's functions and the perturbative Feynman rules as expected. An appendix deals with these and related issues in some detail. It is also assumed that the cosmological constant is zero. This is very important, a non-zero cosmological constant is an obstruction to the long distance behavior of Green's functions and their incumbent Ward-Slavnov-Taylor identities.

Next we apply the strategy given in Eqs. (2.3) and (2.10) to calculate the potential to order  $G^2$ . The basic strategy is to expand the matter action Eq. (2.10) to order  $G$  ( $\kappa^2$ ),

$$S_M = -(M_1 + M_2)T + \sum_{i=1}^2 \left( -\frac{\kappa M_i}{2} \int dt_i h_{00}(x(t_i)) + \frac{\kappa^2 M_i}{8} \int dt_i h_{00}(x(t_i))^2 + \dots \right) \quad (2.17)$$

$$\equiv -(M_1 + M_2)T + \sum_{i=1}^2 \left( J_i \int dt_i h_{00}(x(t_i)) + K_i \int dt_i h_{00}(x(t_i))^2 + \dots \right) \quad (2.18)$$

and expand the partition function in powers of the sources  $J_i$  and  $K_i$ . Next connected graphs in the gravitational sector are retained. The graphs are connected to sources which are vertices for graviton lines. The complete set of graphs contributing to the static potential to order  $\kappa^4$  ( $G^2$ ) is shown in Figs. 1 to 3. The time coordinates of the sources are to be integrated from  $-T/2$  to  $T/2$  and we are left with an effective 3-D theory. The sources of mass  $M_1$  and  $M_2$  are coupled to the gravitational sector at positions  $\vec{0}$  and  $\vec{R}$ . The potential or static energy can be read off from Eq. (2.6) in the large  $T$  limit.

The connected nature of the Green's functions of  $h_{00}(x)$  and local monomials thereof is extremely important and differentiates the potential from its iterations, which are terms of higher order in  $T$  in the expansion of the exponential in Eqs. (2.17), (2.18).

The contribution to lowest order in  $G$  can be computed from the single graviton exchange graph in Fig. 1 [23]. The result is straightforward and familiar

$$f_1 = \left( -\frac{i\kappa M_1}{2} \right) \left( -\frac{i\kappa M_2}{2} \right) \left( -\frac{1}{8\pi^2} \right) \int_{-T/2}^{T/2} dt_1 dt_2 \frac{1}{(t_1 - t_2)^2 - R^2 - i\epsilon} \quad (2.19)$$

Expressing the integrand as a contour integral and integrating gives (in the limit of large  $T$ )

$$f_1 = iT \frac{GM_1 M_2}{R}, \quad (2.20)$$

which, in view of Eq. (2.6), yields the Newtonian potential energy

$$V(R) = -\frac{GM_1M_2}{R}. \quad (2.21)$$

The corrections to order  $G^2$  are of two classes: classical general relativistic (independent of  $\hbar$ ) and quantum mechanical of order  $\hbar$ . They are enumerated in Fig. 2 for the semi-classical contributions and Fig. 3 for the quantum contributions. It is easy to understand that a perturbative expansion of the functional integral in gravity gives us classical contributions, both formally and intuitively. The formal reason is that the expansion includes tree graphs connected to an arbitrarily high number of external classical sources. Boulware and Deser [24] showed that the tree graphs reproduce classical general relativity (see also [25,26]). The intuitive reason is that in gravity there are two dimensionless parameters that lend themselves to a perturbative expansion, one which involves  $\hbar$  and is the ratio of the Planck length with the separation of constituents, and one which is independent of  $\hbar$  and is the ratio of the Schwarzschild radius of a mass with the separation of constituents.

The quantum contributions involve closed loops of gravitons and are of order  $\hbar$ . Fig. 3a is the vacuum polarization and receives contribution from the Fadeev-Popov-Feynman ghost which is a vector particle in gravity; this contribution is well known in the literature and has been calculated long ago.

The calculations are discussed in detail in an appendix. We note the results here. The corrections to the Newtonian potential are denoted by  $\delta V$  and the results are as follows,

$$\delta V_{\text{Cl}}(R) = -\frac{GM_1M_2}{R} \left( \frac{G(M_1 + M_2)}{Rc^2} \left(-1 + \frac{1}{2}\right) \right) \quad (2.22)$$

$$= -\frac{GM_1M_2}{R} \left( -\frac{G(M_1 + M_2)}{2Rc^2} \right) \quad (2.23)$$

from the graphs of Figs. 2a and 2b respectively.

The quantum correction is of the form

$$\delta V_Q(R) = -\frac{GM_1M_2}{R} \left( \frac{G\hbar}{\pi R^2 c^3} \left( \frac{43}{30} + \frac{1}{4} - \frac{5}{6} \right) \right) \quad (2.24)$$

$$= -\frac{GM_1M_2}{R} \frac{17}{20\pi} \frac{G\hbar}{R^2 c^3} \quad (2.25)$$

The detailed numbers come from Figs. 3a, 3b, and 3c respectively. In all of these contributions, there is no difficulty in extracting the large time behavior in the leading static limit for large masses neglecting recoil in Eq. (2.6). Putting everything together gives to order  $G^2$ ,

$$V(R) = -\frac{GM_1M_2}{R} \left( 1 - \frac{G(M_1 + M_2)}{2Rc^2} + \frac{17G\hbar}{20\pi R^2 c^3} \right). \quad (2.26)$$

All other contributions from skeleton graphs to this order are of short range and have support in a region where we do not believe the calculation in any case.

We close this section with some remarks about the regime of validity of our results.

- There is a hierarchy of distance scales in the problem, namely the Compton wavelength of the sources  $\hbar/M_i c$ , the Planck length  $(G\hbar/c^3)^{1/2}$ , and the Schwarzschild radii of the sources,  $2GM_i/c^2$ . These scales are clearly not independent, since the Planck length is the geometric mean of the other two length scales. The corrections to the potential are to be trusted for distances large compared to all of these distance scales. This follows from the explicit expression of the corrections, but most importantly it is implicit in our restriction of staticity. Let us explain this more fully with a familiar Feynman diagrammatic approach. Consider a particle interacting with a massive center through the exchange of a scalar particle (for convenience, not necessity). If the momentum of the exchanged scalar is  $q$  and the initial and final momenta of the scattered particle are  $p$  and  $p'$ , then the scattering amplitude is proportional to the sum of the direct and crossed propagators, namely

$$\frac{i}{(p+q)^2 - m^2 + i\epsilon} + \frac{i}{(p'-q)^2 - m^2 + i\epsilon}. \quad (2.27)$$

In the limit of large mass and assuming on-shell scattering,  $p_0 \approx p'_0 \approx m$ , and the above sum becomes

$$\frac{i}{2mq_0 + i\epsilon} + \frac{i}{-2mq_0 + i\epsilon} = \frac{2\epsilon}{\epsilon^2 + 4m^2 q_0^2} \propto \delta(q_0). \quad (2.28)$$

Numerator factors of  $q_0$  have been neglected in addition to the customary linearization of propagators (Recall that spin 1 and spin two exchange will involve polynomials in  $p + p'$ ). In this limit, one can sum *all* the possible exchange contributions to scattering, in view of the following identity [27]

$$2\pi\delta(\sum q_{i0}) \times \sum_{\text{perms}(q_{10}, \dots, q_{n0})} \frac{i}{q_{10} + i\epsilon} \times \frac{i}{q_{10} + q_{20} + i\epsilon} \times \dots \\ \dots \times \frac{i}{q_{10} + \dots + q_{n-1,0} + i\epsilon} = (2\pi)^n \delta(q_{10}) \times \dots \times \delta(q_{n0}),$$

where the delta function on the left hand-side is the overall energy conserving  $\delta$ -function  $\delta(\sum q_{i0} + p_0 - p'_0) \approx \delta(\sum q_{i0})$ . This identity shows that the sum over the *dynamical* exchanges is equal to a single graph in which the scattered particle field is replaced by a static pointlike classical source  $j(\vec{x}) = \delta(\vec{x}) = \text{F.T.}[2\pi\delta(q_0)]$ . This argument shows that we can neglect the dynamical, quantum behavior of the field which undergoes the scattering, provided that we are at a distance larger than *its* Compton wavelength and larger than distance scales where non-perturbative effects become important. The contribution of powers of  $q_0$  in the numerator will enter at order  $q_0^2$  or higher in accordance with expectations on the low energy behavior of quantum electrodynamics and quantum gravity. It is also clear that these techniques can be generalized to establish a result that allows the factorization and the incumbent exponentiation for any finite correlation of exchanged quanta coupled to the source. This argument, which is not presented in detail here, is useful in establishing the general result in terms of connected Green's functions Eqs. (2.10, 2.11, 2.17, 2.18) for higher polynomials in  $h_{00}$ .

The above argument implies that the corrections to the potential are valid for distances larger than the Compton wavelength of the source, if the mass of the source is less than the Planck mass ( $10^{-5}g$ ) (with recoil corrections of order  $q^2/M_i$ ). This includes all of the elementary particles of the standard model including the hypothetical vector bosons that mediate the baryon number non-conserving interactions of Grand Unified

Theories and other massive particles of broken supersymmetric and technicolor theories. For sources with macroscopic masses, on the other hand, the distance should be greater than the Schwarzschild radius of the source.

- While the main results are similar in spirit to the work of Donoghue [2], the details and results differ. In our work the coupling to static sources Eqs. (2.6) and (2.10) is maintained throughout the entire calculation. In the calculation of Donoghue, the scattering amplitude with recoil and relativistic corrections in the coupling to matter are retained. The potential was defined as the non-relativistic limit of the one particle reducible graphs of the  $t$ -channel, a definition substantially different than the one presented here. An important check on the classical correction to the Newtonian potential obtained here is the complete agreement with the post-Newtonian potential energy of a test particle of mass  $M_2$  in an external field produced by  $M_1$ , in the limit  $M_1 \gg M_2$  and where  $M_2$  has zero velocity, namely  $V = \Phi + 1/2\Phi^2$ , where  $\Phi$  is the Newtonian potential [28]. Our result for the potential energy is also consistent with the expansion of the 00 component of the metric in a Schwarzschild geometry in isotropic coordinates,

$$g_{00} = 1 - \frac{2GM}{R} \left( 1 - \frac{GM}{R} \right) \dots,$$

with the proper post-Newtonian identification  $g_{00} = 1 + 2\Phi + 2\Phi^2$ .

- The quantum corrections in Eq. (2.26) have a definite sign. The physical origin of this sign is due to positivity of the Hilbert space metric over physical states and is discussed in more detail in the appendix. Notice that the Newtonian limit is approached from above and the attractive interaction increases as the distance decreases. This is in agreement with the analogous situation in quantum electrodynamics with the form of the Uehling potential [29].
- A final word of caution is in order. It is only at the one-loop level that the finite long distance behavior can be separated from the divergent small distance counterterms

in an unambiguous way, at least with the field theory techniques presented so far. Non-leading long distance quantum corrections at two loops and higher will receive contributions from divergent higher dimension polynomial counterterms. In order to completely understand the quantum contributions to long range forces in higher orders, one would need an improved definition of the theory. String theory is perhaps a suitable candidate. We return to this question in section III.

## B. Ultra-Relativistic Limit

The study of the elastic forward scattering of two scalar particles at high energies greater than the Planck mass,  $s \gg M_{\text{Pl}}^2 \gg t$ , with  $s = (p_1 + p_2)^2$  and  $t = (p_1 - p_2)^2$ , has received attention in recent years because of its relation to superstring theory.

One of the authors and M. Soldate investigated the problem [3] because of the unitarity difficulty due to the energy growth of the scattering amplitude in any theory of gravity in near the forward scattering. 't Hooft and others [4,5] were interested in the problem because of its possible relation to black hole dynamics, to string theory, and to other field theory contexts. The scattering can be treated nicely by semi-classical methods which go under the name of eikonal approximation or WKB and are well developed in the literature. The basic strategy is to retain the leading terms in the coupling of the external particles to gravitons at high energies and linearize the propagators. Probably the most lucid exposition again comes functional integral techniques which we sketch briefly below. The four-point Green's function for the scattering of four massive scalar particles, interacting only gravitationally, is given by the time-ordered expression

$$\langle 0|T(\phi(x_1)\phi(x_2)\phi(x_3)\phi(0)|0\rangle = \int [Dh] e^{\frac{i}{\hbar} S_{\text{EH}}} G(x_1, x_3; g) G(x_2, 0; g) + \text{perms}, \quad (2.29)$$

neglecting scalar loops, as they do not contribute to power behaved long range contributions. The propagators satisfy in the massless limit (it suffices to retain the massless limit as an approximation in the ultra-relativistic approximation),

$$-\square G(x, x'; g) = \frac{1}{\sqrt{g}} \delta^{(4)}(x - x'), \quad (2.30)$$

where  $\square = g^{-1/2} \partial_\mu (g^{1/2} g^{\mu\nu} \partial_\nu)$  is the generally covariant Laplacian. If the metric is expanded about flat spacetime and the leading terms at high energies are retained, one obtains

$$\left( -\eta^{\mu\nu} \partial_\mu \partial_\nu + \tilde{V}(x) \right) G(x, x') = \delta^{(4)}(x - x'). \quad (2.31)$$

In the leading approximation at high energy, the problem then reduces to potential theory.

The potential is:

$$\tilde{V}(x) = \kappa E^2 h_{--}(x), \quad (2.32)$$

for the right mover and  $(- \rightarrow +)$  for the left mover, where the light-cone coordinates are defined as usual by  $x^\pm = z \pm t$  (see in this context the papers by Kabat and Ortiz [30] and Verlinde and Verlinde [31]). One passes to the amputated proper one-particle propagator through the familiar scattering identities  $G = G_0 + G_0 \Sigma G_0 = G_0 + G_0 \tilde{V} G$ , where  $G_0$  is the free one-particle propagator. Upon transformation to momentum space, one obtains

$$\langle p' | \Sigma | p \rangle = \langle p' | \tilde{V} | \Psi_p \rangle = \int dx e^{-ip' \cdot x / \hbar} \tilde{V}(x) \Psi_p(x), \quad (2.33)$$

where  $\Psi_p(x)$  is the scattering solution which satisfies  $\Psi = \Psi_0 + G_0 \tilde{V} \Psi$  with appropriate scattering boundary conditions. Its WKB scattering solution is

$$\Psi_p(x) = \exp \frac{i}{\hbar} \left\{ p \cdot x - \frac{1}{2E} \int_{-\infty}^{\tau} \tilde{V}(x(\tau')) d\tau' \right\}, \quad (2.34)$$

where  $\tilde{V}(x)$  is given by Eq. (2.32), and the trajectories of the scattered constituents are given in light cone coordinates  $(+, \text{tr}, -)$  by

$$x_1 = (0, \vec{b}, x^-), \quad x_2 = (x^+, \vec{0}, 0). \quad (2.35)$$

In Eq. (2.34),  $\tau$  is a coordinate that parameterizes the trajectory,  $\tau = x^+(x^-)$  for right(left) movers, and  $\vec{b}$  is a 2-vector representing the impact parameter. The on-shell scattering amplitude results from amputation of the Green's function, through use of Eqs. (2.29), (2.33), and (2.34),



$$\mathcal{A}(E, \vec{b}) = 2iE^2 \langle T \exp -\frac{i}{\hbar} \frac{\kappa}{2} \int h_{\mu\nu} T^{\mu\nu} \rangle, \quad (2.36)$$

very much in parallel with Eq. (2.8). The energy momentum tensor in Eq. (2.36) now represents sources for the scattering of two relativistic particles in the so-called shock wave kinematics,

$$T_{\mu\nu} = E \delta^{(3)}(x - x_1(\tau)) \delta_\mu^+ \delta_\nu^+ + E \delta^{(3)}(x - x_2(\tau)) \delta_\mu^- \delta_\nu^-. \quad (2.37)$$

The space-time coordinates in Eq. (2.29) have been simplified by the use of translational invariance and the form of the trajectories, Eq. (2.35) relevant to the scattering kinematics. The first term in Eq. (2.37) represents the right mover at impact parameter  $\vec{b}$  and the second term represents the left mover at the origin in impact parameter space.

A familiar form for the scattering amplitude is derived from Eq. (2.36) after quadratic functional integration to order  $G$  ( $\kappa^2$ ) of the gravitational action. Upon Fourier transformation to the transverse momentum representation, one obtains in the relativistic limit, the familiar eikonal formula with  $\mathcal{A} = \exp i\Delta$ ,

$$\mathcal{M}(E, \vec{q}_{\text{tr}}) = 2iE^2 \int d\vec{q}_{\text{tr}} \exp i\vec{q}_{\text{tr}} \cdot \vec{b} (\exp i\Delta - 1). \quad (2.38)$$

Equation (2.38) is the celebrated eikonal approximation corresponding to the functional average in Eq. (2.36); In lowest order the contribution to the eikonal is given by the two dimensional graviton propagator i.e.  $\Delta \approx (-2E^2 G) \ln \lambda b$ , which integrates to a form obtained in many references [3–5],

$$\mathcal{M} = 8\pi G \frac{s^2}{t} \left\{ \left( \frac{\lambda^2}{t} \right)^{-i\alpha} \frac{\Gamma(1 - i\alpha)}{\Gamma(1 + i\alpha)} \right\} \quad (2.39)$$

where  $\alpha$  is the Regge trajectory for the Newtonian potential ( $\alpha = O(G \frac{s}{\hbar})$ ) as  $s \rightarrow \infty$ ,  $\lambda$  is an infrared regulator for the 2-D effective massless propagator, and  $s$  and  $t$  are the usual Mandelstam variables. Eq. (2.39) is valid for large  $s$  and, near forward scattering, at small  $t$ . The real term in Eq. (2.39) is the purely classical scattering, while the pure phase (bracketed expression) is a contribution quantum mechanical in origin. The latter term corrects the

ill-behaved growth of partial wave amplitudes at high  $s$  and their incumbent violation of unitarity [3].

The general results Eqs. (2.36), (2.38), and (2.39)) have the desirable properties and features:

- Unitarity in impact parameter space is retained at high values of  $s$ . Unitarity follows from reality of the exponential in Eq. (2.40) for elastic scattering and the positivity of its imaginary part for inelastic scattering for radiation of soft gravitons. See in this context the analysis of the CERN group in [19]. They establish a systematic expansion for the analogue of the static semi-classical results in  $G^2 s/b^2$ .
- The reader will see the close relation between the two contexts, ultra-relativistic and static. The ultra-relativistic theory is simply a 2-D image of the static 3-D effective theory.
- The reader will also recall that Eqs. (2.36), (2.38), and (2.39) together with the techniques used in their derivation were the basis of phenomenological models of high energy hadron scattering at least two decades ago [15]. Such techniques produced a geometric picture of high energy diffraction scattering of hadrons.

### III. RELATION TO STRING THEORY AND DISCUSSION

The detailed calculations of the quantum corrections for example are presented only at the one-loop level,  $O(\hbar)$ . It is obvious that in higher orders there will be difficulties. For example, divergent subgraphs by power counting will necessitate higher dimension polynomial valued counter terms coming from operators of dimension four and greater such as  $R^2$ , where  $R$  is the Riemann tensor or any of its contractions. These divergent counterterms can then propagate into the skeletons in calculations analogous to those of Section III. It becomes impossible to proceed further in a theory that is divergent and non-renormalizable such as quantum gravity. Clearly some new definition of the theory is necessary.

The only well developed strategy is superstring theory in one of its various forms. We leave aside the vagaries of reconciling string theory to the phenomenology of particle physics and dwell on the question of a consistent theory of quantum gravity. Fortunately, there are one-loop calculations of on-shell scattering amplitudes in the literature [32,36]. There are even calculations beyond one loop that focus on unitarity which relate the higher loop graphs to tree graphs. The calculations are technical and involve the moduli space of higher genus Riemann surfaces, the general name given to certain complex manifolds that appear in string perturbation theory. The calculation of the one-loop amplitude by Montag and Weisberger [32] is perhaps the most relevant, and we will quote and interpret their results for our purposes.

In string theory another scale emerges; namely the Regge slope or string tension  $\alpha'$  which plays two roles: It sets the scale of the coupling to and the mass scale for the higher mass states in the string spectrum of states. The field theory limit is then achieved by taking the  $\alpha' \rightarrow 0$  limit.

Higher-order graphs in string theory (of which there are fewer due to duality, *e.g.* one at the one loop level) involve integration over complex coordinates in contrast to the integration over momenta of the corresponding graphs in field theory.

To order  $\kappa^2$ , the structure of the tree plus one-loop, on-shell, invariant scattering amplitude for massless string constituents *i.e.* dilatons or gravitons is

$$\mathcal{M}(s, t, u) = K[\mathcal{A}_0(s, t, u) + c\mathcal{A}_1(\alpha'; s, t, u)], \quad (3.1)$$

where  $K$  is a kinematical factor to ensure gauge invariance and proper mass-shell behavior, and  $c$  is a dimensionless constant. In Eq. (3.1),  $\mathcal{A}_0$  is the typical Veneziano-type amplitude,

$$\mathcal{A}_0(s, t, u) = \frac{1}{stu} \frac{\Gamma(1 - \frac{\alpha'}{4}s)\Gamma(1 - \frac{\alpha'}{4}t)\Gamma(1 - \frac{\alpha'}{4}u)}{\Gamma(1 + \frac{\alpha'}{4}s)\Gamma(1 + \frac{\alpha'}{4}t)\Gamma(1 + \frac{\alpha'}{4}u)}, \quad (3.2)$$

while  $\mathcal{A}_1(\alpha'; s, t, u)$  is the one-loop string amplitude, the form of which is not essential for our purposes (it is given by an integral of certain world-sheet vertex operators over the moduli space of a torus). It suffices to note that in the low energy limit, the one-loop amplitude

can be expanded in powers of  $\alpha'$  and the field theory limit is recovered in the limit  $\alpha' \rightarrow 0$ , as long as  $|\alpha's|, |\alpha't|, |\alpha'u| \ll 1$ . The expansion is non-trivial due to subtleties in the region of integration in the moduli space which ensure a finite amplitude in the physical region of  $s, t, u$ . Nevertheless, when the dust settles, the result that emerges can be written as an expansion, where the first non-trivial term is  $O(\alpha'^2)$  due to symmetric integration in moduli space is

$$\mathcal{A}_1(\alpha'; s, t, u) = \mathcal{A}_1^{(0)}(s, t, u) + \alpha'^2 \mathcal{A}_1^{(2)}(s, t, u) + \dots, \quad (3.3)$$

where  $\mathcal{A}^{(0)}$  and  $\mathcal{A}^{(2)}$  are the first two terms in the expansion in  $\alpha'$ . The first term  $\mathcal{A}^{(0)}$  is the zero slope or field theory limit, and the next term is a local polynomial in the momenta at low energy, i.e.  $\mathcal{A}^{(2)} = O(s^2, t^2, u^2)$ . The field theory amplitude has a representation in terms of a Feynman parameter integral,

$$\mathcal{A}_1^{(0)} = \int \prod_i d\beta_i \delta\left(1 - \sum_j \beta_j\right) \phi(s, t, \beta)^{(10-d)/2} + \text{sym.}, \quad (3.4)$$

where  $\phi(s, t, \beta) = \beta_1\beta_2s - \beta_3\beta_4t$  is the usual polynomial in momentum invariants,  $d$  is the number of compactified dimensions ( $d = 6$ ), and there is to be a symmetrization with respect to the interchanges  $s \rightarrow t$ ,  $s \rightarrow u$ , etc. At low energy the massive states are completely decoupled and the field theory contributions completely dominate the amplitude. Hence it is seen that the field theory contributions overshadow the string corrections at low energies (large distances), as expected also from the effective action for the massless modes of the string (however, for a possible low-energy window on string-scale physics see [33]).

Although the actual calculation presented here on the first non-trivial loop correction is clearly suggestive, we expect that higher order string calculations will support the general result of a gauge-invariant amplitude with polynomial corrections to the field theory result at low energies. At small distances, and  $s, t$ , and  $u$  large and comparable, the situation is quite different and string considerations are important and essential [34,35].

We close this section with some further remarks concerning the relation of our work on the long range forces in quantum gravity with some related issues in gauge theories.

Namely, dynamical mechanisms of spontaneous symmetry breaking and mass generation render the Coulomb potential to be the only leading long range force in electrodynamics between charged objects. The finite fermion mass shields all long range forces at the quantum level in such theories. In non-abelian gauge theories such as QCD, confinement presumably renders all long range forces inoperable, once the hadron mass spectrum is achieved in a realistic calculational framework.

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### APPENDIX A:

In this appendix we present some of the details of the calculations. The Feynman rules in de Donder gauge have been presented in the literature. We will present what is sufficient for the calculations at hand. The expansion of the metric Eqs. (2.5) and (2.10) give us all of the coupling to the matter sources needed. The one- and two- $h_{00}(x)$  graviton vertices that couple to the matter sources are simply  $(-\kappa i M_j/2)$  and  $(i\kappa^2 M_j/4)$ , respectively. Furthermore, we need the triple graviton vertex. Actually, since we require the three- $h_{00}$  Green's function, we only need some special projections of the full triple vertex. One can show that the

momentum space transform of the configuration space three-point function involving only  $h_{00}$ 's is given by the projection

$$V_3 = V_{000000} - \frac{1}{2}(V_{0000}{}^\mu{}_\mu + V_{00}{}^\mu{}_\mu{}_{00} + V^\mu{}_\mu{}_{0000}) + \frac{1}{4}(V_{00}{}^\mu{}_\mu{}^\nu{}_\nu + V^\mu{}_\mu{}^\nu{}_\nu{}_{00} + V^\mu{}_\mu{}_{00}{}^\nu{}_\nu) - \frac{1}{8}V^\mu{}_\mu{}^\nu{}_\nu{}^\lambda{}_\lambda, \quad (\text{A1})$$

where  $V_{\kappa\lambda\mu\nu\rho\sigma}(p, q, -(p+q))$  is the full triple graviton vertex in momentum space. The projected vertex is

$$V_3(p, q) = -\frac{i\kappa}{4} \left( p^2 + q^2 + (p+q)^2 - 2(p_0q_0 - (p_0+q_0)^2) \right), \quad (\text{A2})$$

These vertices are sufficient to compute the static potential to order  $G^2$ . All contributions are relatively straightforward to compute except for the vacuum polarization of Fig. 3. All computations are performed in the harmonic gauge of Eq. (2.14). The contribution to the connected Green's function of Eqs. (2.17) and (2.18) from the graph in Fig. 2a is

$$f_{2a} = \frac{i\kappa^3}{16} M_1 M_2 (M_1 + M_2) \int dt_1 dt_2 dt_3 G(t_1, \vec{0}; t_2, \vec{0}; t_3, \vec{R}). \quad (\text{A3})$$

where  $G(x_1, x_2, x_3)$  is the three-point function of the temporal gravitons. Since all three gravitons are connected to static sources, the 0 components of their momenta vanish, and we obtain that the three-point function in this case is the product of two propagators. The time integrals are trivial delta functions of  $p_0$  and  $q_0$ , times another delta function  $2\pi\delta(0) = T$ . Two of the three contributions to the final Fourier integral which transforms back to configuration space are divergent. However the divergence is a ultra-local divergence, which is irrelevant to the long-distance effects. The finite contribution is

$$f_{2a} = \frac{i\kappa^4}{64} M_1 M_2 (M_1 + M_2) T \left( i \int \frac{d^3\vec{p}}{(2\pi)^3} e^{-i\vec{p}\cdot\vec{R}} \frac{1}{p^2} \right)^2 \quad (\text{A4})$$

$$= -iT \frac{GM_1 M_2}{R} \frac{G(M_1 + M_2)}{R}. \quad (\text{A5})$$

Fig. 2b contributes

$$f_{2b} = \frac{1}{2} \left( -\frac{i\kappa}{2} \right)^2 \left( \frac{i\kappa^2}{4} \right) M_1 M_2 (M_1 + M_2) \int dt dt_1 dt_2 \Delta(t - t_1, \vec{R}) \Delta(t - t_2, \vec{R}). \quad (\text{A6})$$

A straightforward evaluation of the integral in Eq. (A.6) yields the result

$$Re \int_{-\frac{T}{2}}^{\frac{T}{2}} dt dt_1 dt_2 \frac{1}{(t-t_1)^2 - R^2 - i\epsilon} \frac{1}{(t-t_2)^2 - R^2 - i\epsilon} = -\frac{\pi^2 T}{R^2} + \text{subleading}. \quad (\text{A7})$$

This gives

$$f_{2b} = iT \frac{GM_1 M_2}{R} \frac{G(M_1 + M_2)}{2R}. \quad (\text{A8})$$

The sum of  $f_{2a}$  and  $f_{2b}$  gives rise to the post-Newtonian correction to the potential.

Next, we consider the quantum corrections. First, the polarization contributes

$$P_{00\alpha\beta} \Pi^{\alpha\beta\gamma\delta} P_{\gamma\delta 00} = \frac{\kappa^2}{32\pi^2} q^4 \left( \frac{43}{120} \right) (-\ln(-q^2)). \quad (\text{A9})$$

The vacuum polarization Fig. 3a receives contribution from both the graviton loop and the Fadeev-Popov-Feynman-Mandelstam ghost loop; this contribution has been computed more than two decades ago by Capper et al [36] and Duff [37], and by 't Hooft and Veltman [38]. 't Hooft and Veltman obtain general results that can be transcribed to the long range potential that we are computing. The result can be written in terms of coordinate invariant quantities quadratic in the Ricci tensor and the curvature scalar. Donoghue [2] has quoted their result in the form given below. We have not checked the complete calculation of the vacuum polarization from first principles. Recalling that the propagator is corrected as

$$\frac{i}{q^2} P_{0000} + \frac{i}{q^2} P_{00\alpha\beta} i \Pi^{\alpha\beta\gamma\delta} \frac{i}{q^2} P_{\gamma\delta 00} = i \left( -\frac{1}{2\bar{q}^2} + \frac{G}{\pi} \frac{43}{120} \ln \bar{q}^2 \right), \quad (\text{A10})$$

the vacuum polarization contribution to the quantum corrections follows.

Fig. 3b is calculated easiest in configuration space,

$$f_{3b} = \frac{1}{2} \left( \frac{i\kappa^2}{4} \right)^2 M_1 M_2 \left( -\frac{1}{8\pi^2} \right)^2 \int dt_1 dt_2 \frac{1}{((t_1 - t_2)^2 - R^2 - i\epsilon)^2}. \quad (\text{A11})$$

One notes that the square of the propagator is proportional to the derivative of a single propagator with respect to  $R$ . Recalling (2.19) and (2.20), we obtain

$$f_{3b} = iT \frac{GM_1 M_2}{R} \frac{G}{4\pi R^2}. \quad (\text{A12})$$

The quantum correction of the graph in Fig. 3c involves the same triple graviton vertex and calculates to

$$f_{3c} = -T \frac{\kappa^4 M_1 M_2}{32} \int \frac{d^3 \vec{p}}{(2\pi)^3} e^{-i\vec{p} \cdot \vec{R}} \int \frac{d^4 q}{(2\pi)^4} \left( \frac{1}{q^2(p+q)^2} + 2 \frac{q_0^2}{p^2 q^2 (p+q)^2} \right) \Big|_{p^0=0}. \quad (\text{A13})$$

Since

$$\int \frac{d^4 q}{(2\pi)^4} \frac{q_0^2}{p^2 q^2 (p+q)^2} \Big|_{p^0=0} = -\frac{p^2}{12} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2(p+q)^2} \Big|_{p^0=0} \quad (\text{A14})$$

$$= -\frac{p^2}{12} \left( -\frac{i}{16\pi^2} \ln(-p^2) + p\text{-independent} \right), \quad (\text{A15})$$

as can be shown in dimensional regularization, one obtains the advertised result, after discarding unimportant short-range divergences. It is useful to note that

$$\int \frac{d^3 \vec{p}}{(2\pi)^3} e^{-i\vec{p} \cdot \vec{R}} \ln p^2 = -\frac{1}{2\pi R^3}. \quad (\text{A16})$$

Notice that other surviving zero mass non-gravitational contributions i.e. photons, massless neutrinos etc. will modify the results of this appendix, namely the vacuum polarization loop Eq. (A.10), and produce additive contributions as expected. Massive fields will of course produce forces of finite range which do not survive at large distances.

Notice also that the quantum corrections have a sign that increases the attractive gravitational interaction between massive sources as the separation between constituents is decreased. This sign has a physical origin. Namely the long range forces can be traced to real parts of amplitudes with positive definite absorptive parts coming from physical intermediate states in the crossed channel which is the source of the logarithms; See Eq. (A.15), for example. Unphysical polarizations in such intermediate states are cancelled by ghost loops as required by unitarity. Hence, positivity and the associated Schwarz inequality guarantee the definite sign of the three contributions of Fig. 3. While this is similar to quantum electrodynamics whose vacuum is dielectric in nature with dielectric constant greater than one; it is totally at variance with QCD. In QCD contributions purely real in origin reverse the sign of coupling constant renormalization and produce the inevitable asymptotic freedom and its associated paramagnetic vacuum.



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**Fig. 1**

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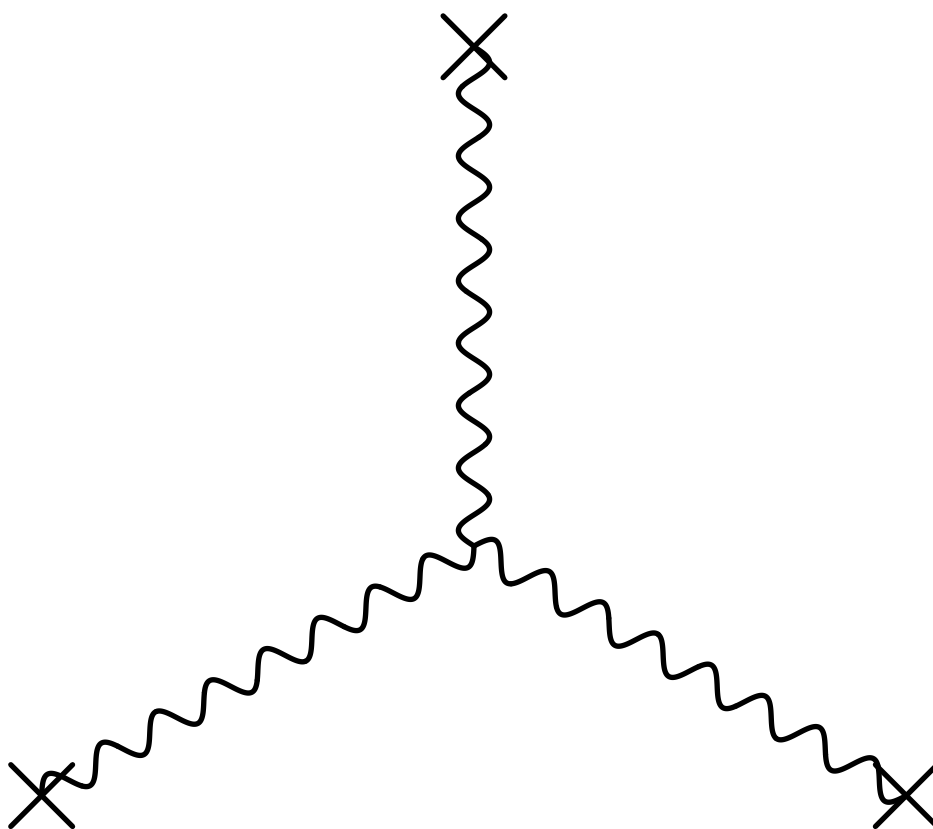


Fig. 2a

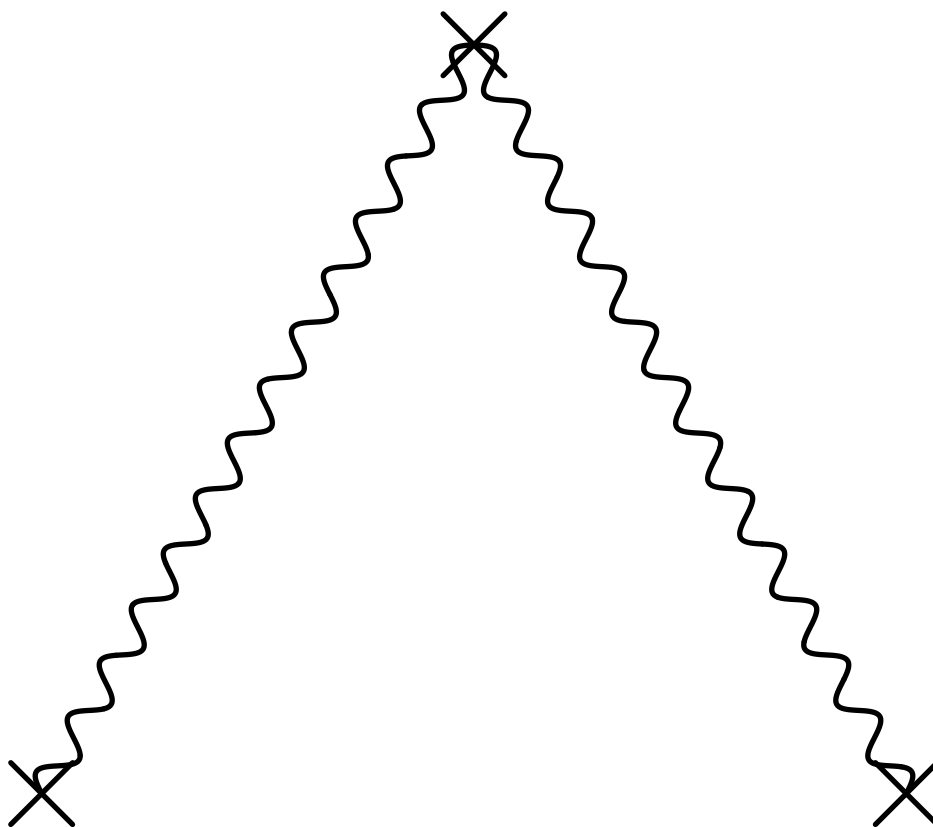


Fig. 2b

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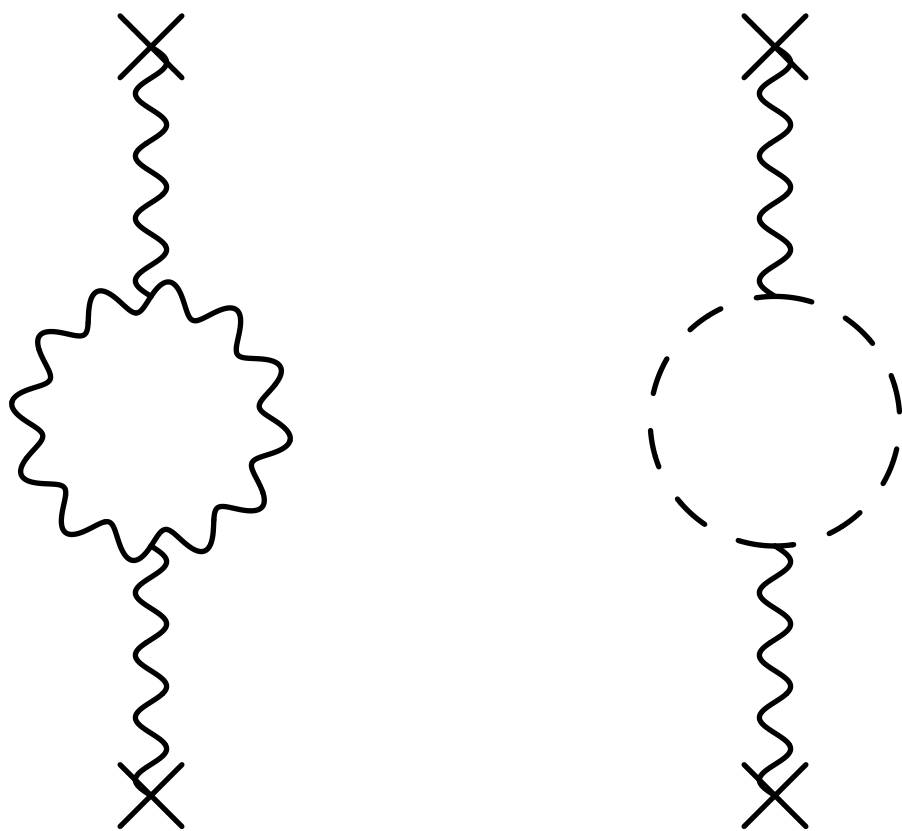


Fig. 3a

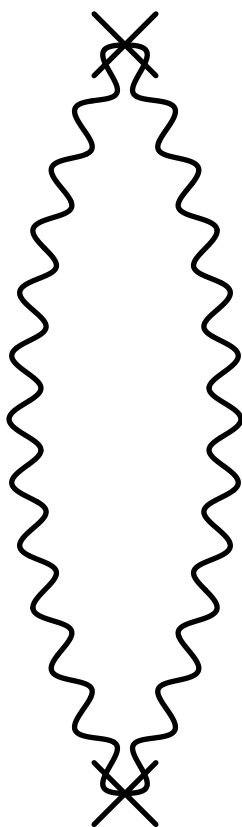


Fig. 3b

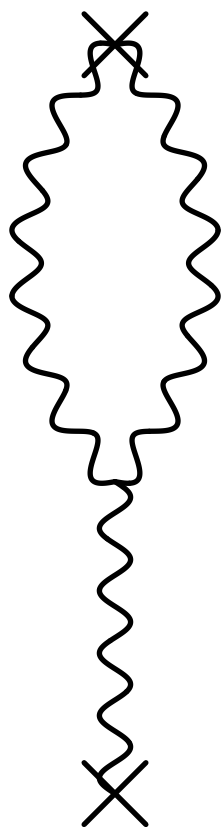


Fig. 3c

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